

## Application Note #5452

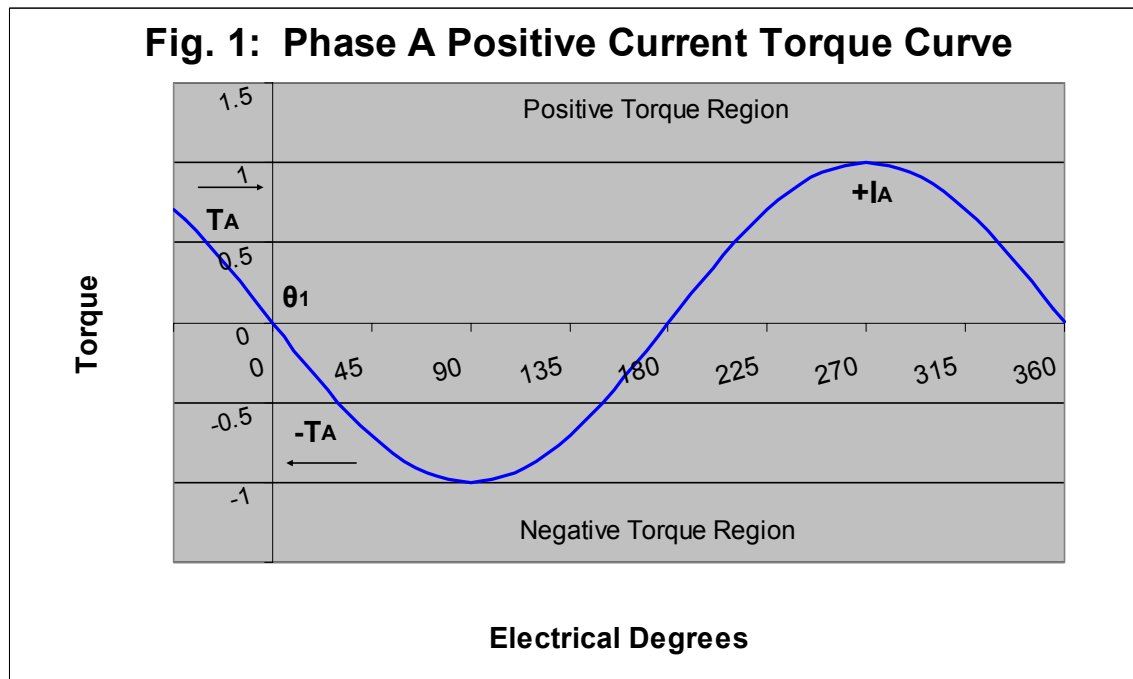
### Microstepping Performance: Resolution, Accuracy & Repeatability

#### Introduction

Microstepping is a method for controlling stepper motors. It offers improved performance by providing finer position steps, reduced vibration, and smoother motion. However, microstepping has limitations on performance. We will discuss these limitations, and how to analyze them to predict realistic microstepping performance. Let us first start with the control modes of stepper motors.

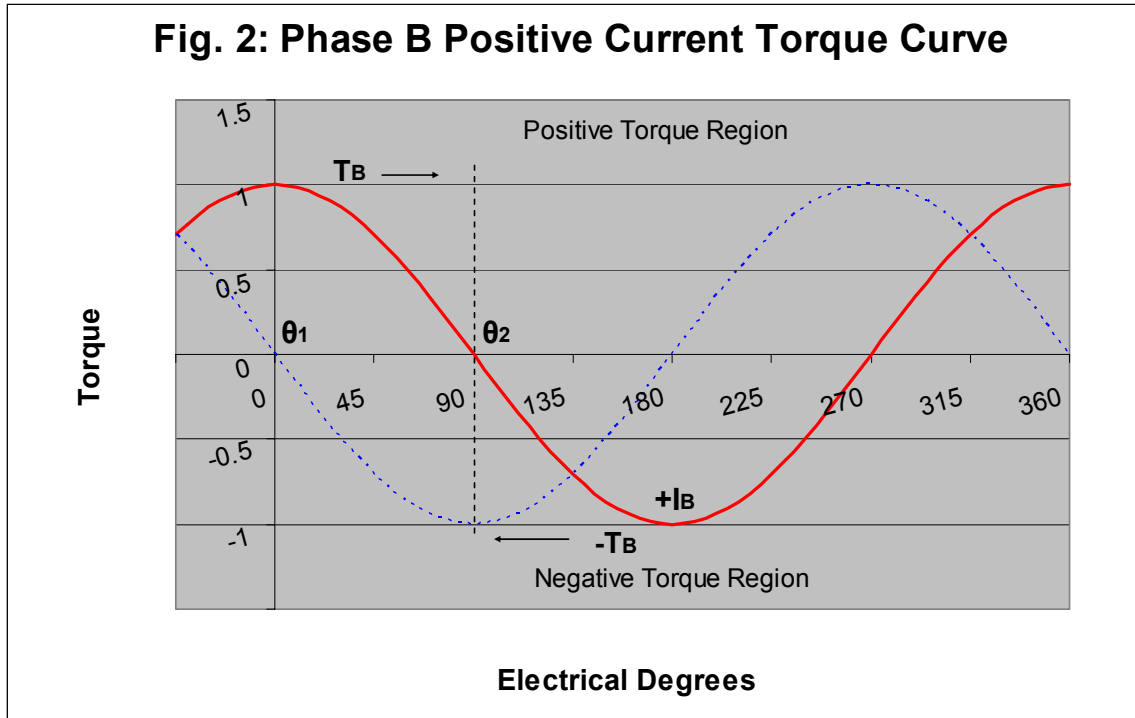
#### Full Step Mode

When we apply a positive current through Phase A, it produces a torque. Fig.1 shows the torque vs. position in electrical degrees.



If the motor is positioned less than position  $\theta_1$  it sees a positive torque and moves forward. If the motor begins at a position greater than  $\theta_1$  it sees a negative torque and moves back. In either case the motor moves to  $\theta_1$ , the stable equilibrium point of phase A with positive current flow. To move the motor

forward we switch off Phase A and activate Phase B with a positive current. The resulting move from  $\theta_1$  shown in Fig.2, is the equilibrium point of Phase B,  $\theta_2$ .



To move the motor continuously, we then switch current off Phase B and back onto Phase A with a negative current. This inverts the torque curve profile of Phase A from Fig. 1, creating forward torque at the previous equilibrium position,  $\theta_2$ . This effect keeps the motor moving to the next equilibrium position,  $\theta_3$ .

**Fig. 3: Phase A Negative Current Torque Curve**

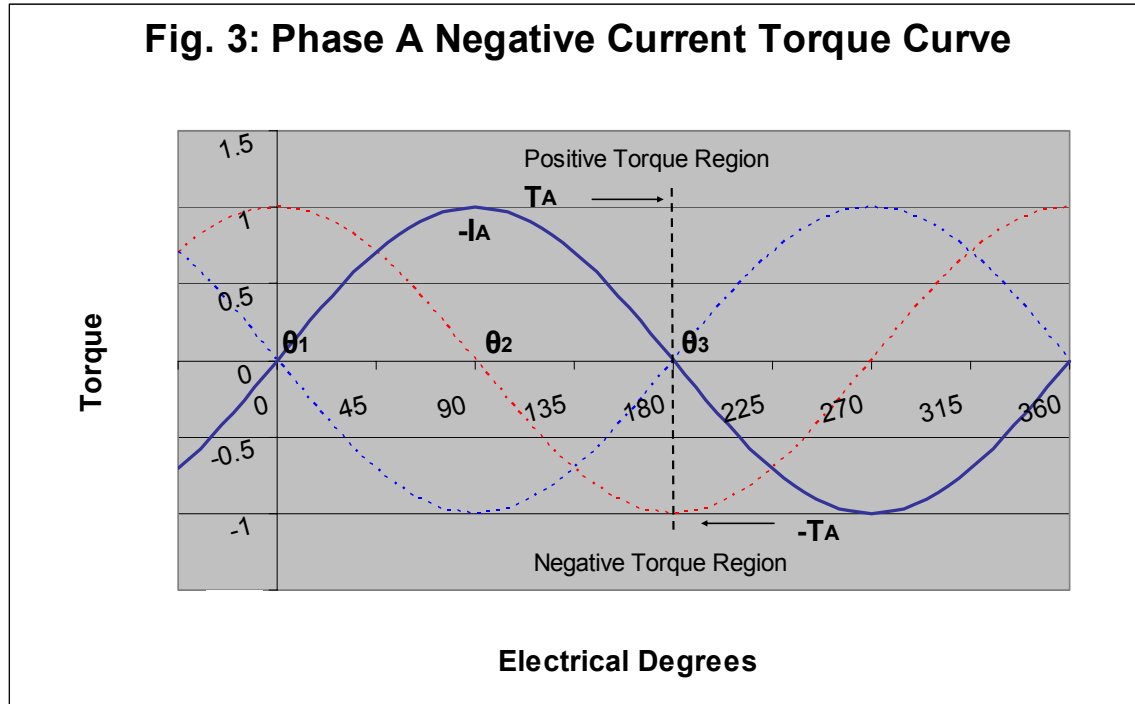


Fig. 3 superimposes the negative current flow for Phases A along with the previous positive current flows and the associated forward motion equilibrium positions  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ . The reverse direction for the motor is executed in the same manner, by simply following Fig. 3 from right left to switch the current on for reverse torque at each equilibrium position. This current switching control mode is known as the Full Step mode.

Full Step Current Conditions:

$$I_A = \pm 1 \ \& \ I_B = 0$$

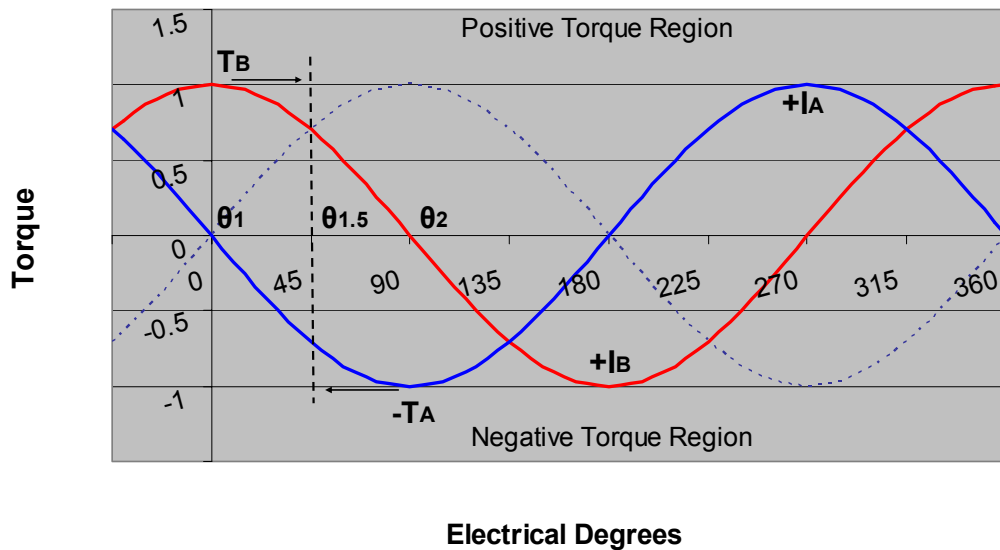
$$I_A = 0 \ \& \ I_B = \pm 1$$

But what about the intermittent positions between these equilibrium positions. Can we go there as well? This is where the Half Stepping control mode comes in.

### **Half Step Mode**

Half stepping requires switching current on to both phases at the same time, effectively halving the step angle.

**Fig. 4: Phase A & B Positive Current Torque Curves**



In Fig. 4 we see that Phase A positive current and of Phase B positive current cause the motor to stop in-between  $\theta_1$  &  $\theta_2$ . This half step position  $\theta_{1.5}$  results from the sum of the opposing torques.

$$-T_A + T_B = 0$$

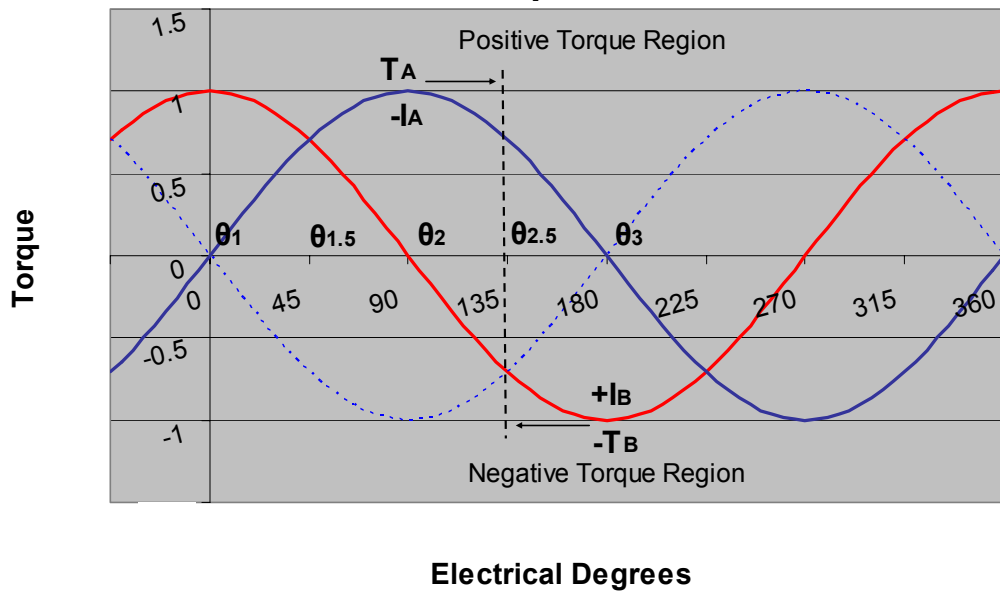
Halfstep Position:  $\theta_{1.5}$

To continue halfstepping forward we turn off Phase A to allow  $\theta_2$  position (as previously defined in Fig. 2), and then switch on negative current to Phase A. In Fig. 5, Phase B positive current is causing backward torque toward full-step equilibrium  $\theta_2$ , yet Phase A negative current is causing forward torque to the next full step equilibrium  $\theta_3$ . The next halfstep position  $\theta_{2.5}$  results from the sum of the opposing torques.

$$T_A + -T_B = 0$$

Halfstep Position:  $\theta_{2.5}$ .

**Fig. 5: Phase A Negative, Phase B Positive Current Torque Curves**



Halfstep Current Conditions:

$$I_A = 1 \text{ \& } I_B = 1$$

$$I_A = -1 \text{ \& } I_B = 1$$

$$I_A = -1 \text{ \& } I_B = -1$$

$$I_A = 1 \text{ \& } I_B = -1$$

### ***What is Microstepping?***

So what about further step angle resolution? If the current is continuously controlled, then positions in-between half-step positions are possible. This is the essence of the microstepping control mode. It allows smaller step angles and also results in smoother motion at lower speeds, reduced torque ripple and less vibration. To find out the motor position in microstepping mode, we need to run the following analysis.

First note that the torque curves of Fig. 1 and Fig. 2 can be expressed as

$$T_A = -K I_A \sin(\theta) \quad (\text{Eq.1a})$$

and

$$T_B = K I_B \cos(\theta) \quad (\text{Eq.1b})$$

where K is a torque constant associated with the specific motor.

The motor will move to the point where the net torque is zero, or, where

$$T_A + T_B = T_g = 0 \quad (\text{Eq.2})$$

Thus,

$$-K I_A \sin(\theta) + K I_B \cos(\theta) = 0$$

Dividing by K leads to

$$-I_A \sin(\theta) + I_B \cos(\theta) = 0$$

Which can be written as

$$\tan(\theta) = I_B / I_A$$

Leading to the position

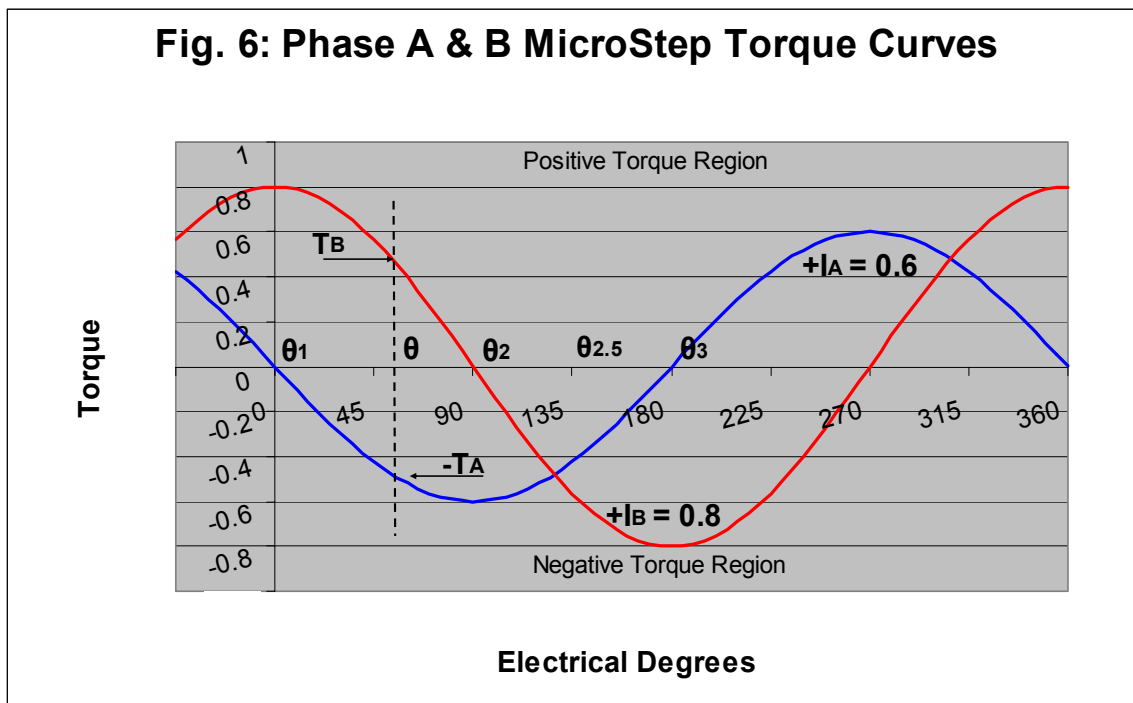
$$\theta = \tan^{-1}(I_B / I_A) \quad (\text{Eq. 3})$$

To illustrate the analysis, consider the case where the motor currents are

$$I_A = 0.6 \text{ A}$$

$$I_B = 0.8 \text{ A}$$

According to Eq. 3, the resulting position is  $53.16^\circ$ .



At this position, the torque generated by Phase A is opposed by the torque of Phase B, with a net torque of zero. By changing  $I_A$  and  $I_B$  we can reach intermediate positions of a very fine resolution.

Now that we have refreshed our background of the control modes of stepper motors, let's turn our attention to the main topic of the discussion, the realistic performance of the microstepping.

### **Performance**

Resolution, Accuracy, and Repeatability describe microstepping performance.

**Resolution** is the commanded position increment.

**Accuracy** is the absolute position error relative to commanded position.

**Repeatability** is the relative position error over many commanded positions.

To clarify the terms, let's consider a sample system of a drive and motor with a resolution of 12,800 microsteps/revolution, or 35.6 counts per degree. From here on, counts will be synonymous with microsteps.

Commanded move from 0 is 1000 counts:  $\theta_o = 1000$

Actual motor position ends up at 950 counts:  $\theta_A = 950$

After repetition of the two moves, the position is observed to

fluctuate 5 counts on either side of the actual positions:  $\theta_{AR} = 945 - 955$

Therefore,

Resolution = 35.6 counts/degree

Accuracy = +/- 50 counts

Repeatability = +/- 5 counts

In this system, which is true for all systems, resolution does not guarantee actual position accuracy. The true performance of a step motor and drive is reduced by many sources of error. The following table consolidates the performance errors into general sources.

**Sources of Error Affecting Performance**

<b>Source</b>	<b>Affects Accuracy</b>	<b>Error is Repeatable</b>
Total Static Friction	Yes	No
Motor Torque Harmonics	Yes	Yes
Drive Bias	Yes	Yes
Quantization Effects	Yes	Yes

**Table 1. Sources of Position Error**

### **Two Types of Error: Non-Repeatable & Repeatable**

There are two types of errors, those that are non-repeatable and those that are repeatable. Non-repeatable errors are unpredictable, within a range, and have the greatest affect on accuracy. Repeatable errors produce predictable, repeatable effects upon accuracy.

#### **Non-Repeatable**

Non-repeatable position error is known as Absolute Position Error. Looking to Table 1, the total static friction of the motor and load affects accuracy, and the position error is not repeatable. This is due to the inherent variability of the torque induced by friction. It is important to understand that this error cannot be improved upon by motor or drive resolutions that are better than the magnitude of the error. The following is a brief derivation of the equation used to calculate this error.

Current is controlled to the phases by the following equations

$$I_A = I_o \text{ Cos}(\theta_C) \quad (\text{Eq. 4a})$$

$$I_B = I_o \text{ Sin}(\theta_C) \quad (\text{Eq. 4b})$$

where  $\theta_C$  is the commanded position in electrical degrees and  $I_o$  is the nominal current. By substituting Eq. 4a & 4b into Eq. 1a & 1b and then the into Eq. 2

$$T_g = K I_o [-\text{Cos}(\theta_C) \text{ Sin}(\theta_A) + \text{Sin}(\theta_C) \text{ Cos}(\theta_A)] \quad (\text{Eq. 5})$$

where  $\theta_A$  is the actual position in electrical degrees. Applying trigonometric identity to Eq. 5, the torque of friction is

$$T_F = K I_o \text{ Sin}(\theta_C - \theta_A) \quad (\text{Eq. 6})$$

Then, the calculated position error due to friction  $(\theta_C - \theta_A) = \theta_{NRE}$  is,

$$\theta_{NRE} = +/- [\text{Sin}^{-1}(T_F/K I_o)] \text{ (electrical degrees)} \quad (\text{Eq. 7})$$



Resolution = 12,800 counts/revolution = 0.028° per count  
Torque of Static Friction(Motor & Load),  $T_F = 5$  oz-in

### ***Non-Repeatable Error***

Commanded move is 1000 counts

$$\theta_o = 1000 = 28.125^\circ (\text{Mechanical Degrees})$$

From Eq. 7 for Static Friction Error

$$\theta_{NRE} = +/-[\text{Sin}^{-1}(T_F/KI_o)] = +/- 8.21^\circ (\text{Electrical Degrees})$$

Converting Electrical Degrees to Mechanical Degrees

$$1 \text{ Step} = 360^\circ/200 \text{ steps/rev} = 1.8^\circ (\text{Mechanical Degrees})$$

$$1 \text{ Step} = 1.8^\circ = 90^\circ (\text{Electrical Degrees})$$

$$1.8^\circ/90^\circ = 0.02 (\text{Mechanical Degrees per Electrical Degree})$$

Therefore,

$$\theta_{NRE} \times 0.02 = +/-0.164^\circ (\text{Mechanical Degrees})$$

The sum of Non-Repeatable Error with resolution of 0.028° per count is

$$\theta_{NRE} = 0.164^\circ/0.028^\circ = +/- \mathbf{5.9 \text{ counts error}}$$

Thus, due to non-repeatable error alone, there is an absolute position error of 6 counts to be considered.

### ***Repeatable Error***

Commanded move is 1000 counts

$$\theta_o = 1000 = 28.125^\circ (\text{Mechanical Degrees})$$

Due to non-repeatable error, the absolute position was evaluated as

$$\theta = 28.125 +/- 0.164^\circ \text{ or } 27.96^\circ - 28.29^\circ$$

By empirical evaluation the repeatable relative position is between 27.90° and 28.35°.

Repeatable Actual Motor Position:  $\theta = 27.90^\circ$  and  $28.35^\circ$ .

Repeatable Position Error:  $\theta_{RE} = +/- 0.06^\circ$  (Mechanical Degrees)

And, with resolution of 0.028° per count

$$\theta_{RE} = 0.06^\circ/0.028^\circ = +/- \mathbf{2.1 \text{ counts error}}$$

Therefore, due to repeatable sources of error, there is an additional 2 counts of position error to be considered. Overall, the accuracy of the system is within +/-6 counts, with a repeatability of +/-2 counts, not considering any axis mechanics external to the motor and drive. This indicates that even though the system has a resolution of 12,800 counts or,  $0.028^\circ$  per count, the actual position could be +/-8 counts(microsteps) around the commanded position.

### **Conclusion**

When considering the performance of an application, it is important to understand what the performance goal is. Some applications require absolute positional accuracy, whereas others are less concerned with accuracy and more with repeatability. It is equally important to consider the positional effects of other mechanical components associated with each axis. Lead screws, couplings and other mechanics will add to the repeatable error through coupling non-linearity, and to the non-repeatable error through tolerance.

The microstepping control method offers improved performance potential for stepper motor axes. However, resolution is no guarantee of actual position accuracy. Each axis has a variety of individual error sources which should be quantified and evaluated to predict actual microstep performance. For more information on step motor control, please see the following links.

#### Monitoring Step Motor Operation

<http://galilmc.com/support/appnotes/optima/note2417.pdf>

#### Backlash Compensation with Closed Loop Steppers

<http://66.60.181.132/xnet/pdfcheckreg.aspx?location=servotrends&target=closedloopstepper>